AIL has had a deep interest in traffic control since the founding of the Company in 1945. For the most part, this interest has been in the problems of air traffic control and Warren D. White, Engineering Consultant at AIL, has been deeply involved in this work. In this however, the seventh article in our current series, we present the results of a study resulting from Warren's travels to and from the laboratory rather than his work on the premises. The simple result he obtains regarding highway traffic illustrates the close connection between electronics and our ordinary daily life.

Backward Waves in Highway Traffic Jams

From time to time our engineers become interested in a variety of problems which seem often remote from our main line of business. Some of these problems are not likely to interest a substantial cross section of readers of the Proceedings of the IRE, but we believe that the subject of this note is familiar to all. The simple calculation which follows may inspire some engineer caught in a traffic jam to experiment with his car and check the accuracy of the theory.

The problem that is being studied is that of the flow of cars in a heavily traveled highway. The steady state solution has been studied by several people interested in determining the capacity of a highway as a function of speed. We are not going to discuss this side of the problem despite the fact that many interesting facts would emerge from this study. We would like to show you today one of the reasons for the stop-and-go oscillatory behavior of car flow and the related existence of backward waves in the "car plasma."

Assume a string of cars proceeding single file such that each car can see only the car in front of him. Assume that traffic is such that passing by changing to another lane is impractical. Let X_i represent the position of the ith car and X_{i-1} represent the position of the car in front of the ith car. Now we further assume that the desired steady state condition is that the spacing between cars should be proportional to speed, that is

$$X_{i-1} - X_i = \tau\dot{X}_i$$

Although this is the condition desired under steady state conditions, it cannot be maintained precisely under changing or transient conditions since the driver of the ith car has only finite response capabilities. We therefore assume that the acceleration of the ith car is proportional to the deviation from the desired norm, i.e.,

$$\ddot{X}_i = \kappa(X_{i-1} - X_i - \tau\dot{X}_i)$$

or

$$\ddot{X}_i + \kappa\tau\dot{X}_i + \kappa X_i = \kappa X_{i-1}$$

We may now consider the two cars as a coupled system in which the input is the motion of the $(i-1)$th car and the output is the motion of the ith car. The frequency response of the system for sine wave input is

$$|F(j\omega)| = \frac{\kappa}{\sqrt{\kappa^2 - 2\kappa\omega^2\left(1-\frac{\kappa\tau^2}{2}\right) + \omega^4}}.$$

If

$$\kappa\tau^2 = 2$$

this system has a maximally flat response like that of critically coupled tuned circuits. On the other hand, if $\kappa\tau^2 < 2$ the system will have double peaked response like that of overcoupled tuned circuits and at some frequency the response will be greater than unity. In other words, the system will have positive gain. If now we consider a long string of cars, there may be only a small gain in each car-to-car link but the overall gain can be quite large. This means that if one driver slows up momentarily to ease an itch in his foot, he will excite a wave that will be amplified as it propagates back down the line of cars until at some point maybe a mile or so back, the modulation of forward velocity will reach 100% and cars will come to a dead stop for no apparent reason.

It will be noted that this situation could be cured by increasing τ or κ or both. Increasing τ corresponds to reducing the traffic rate since $1/\tau$ cars pass a given point in a unit time under steady state conditions. This of course can be done either by eliminating some of the people who drive cars or building more roads so that there are less cars on each road. Increasing κ corresponds to increasing the natural response frequency of the driver. It could be accomplished by providing more anticipation in the system. For example, where the highway goes around a curve so that the driver can see several cars ahead, it may be noticed that wave propagation is damped appreciably. We might achieve the same result by installing periscopes on every car or by providing small radar sets to measure closing rate directly. Neither of these methods appear likely to find mass acceptance but an improvement on the stoplight signal so that it gives a better indication of the true acceleration or deceleration of the car ahead might provide a practical approach to the problem.

Of course a complete theory of automotive traffic motion would be much more complicated and would include the fact that some drivers like to go faster than others, that some passing is possible even under heavy traffic conditions, etc. The development of such a theory would require considerable continued effort on the part of a number of people.

We wonder whether some simple useful suggestions may not be reached if this problem is considered more in detail. Even if this advertisement is not successful in getting more theoretical work on these questions we hope that it will be of some help in suggesting a variety of experimental tests. This company takes no responsibility however for the tickets that antiprogressive traffic policemen may hand to our scientific minded readers.

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